Physics 237b: LISA Lasers and Optics

http://huey.jpl.nasa.gov/~respero/lisa

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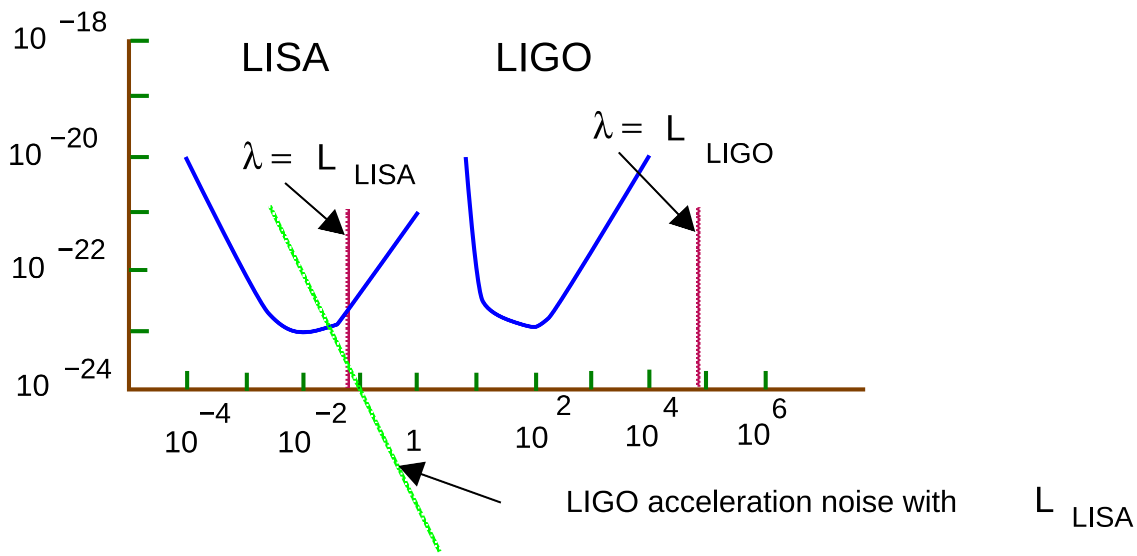
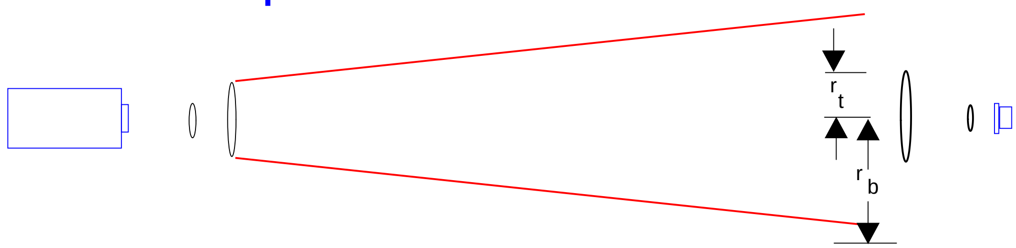
Antenna size compared to signal wavelength



- $L = 5 \cdot 10^9\,\mathrm{m}$ corresponds to frequency $f_L = c/L = 0.06\,\mathrm{Hz}$, near the middle of LISA's sensitive band.

- LIGO, with $f_L = 75\,\mathrm{kHz}$, operates in the long-wave limit.

LISA operates in the far-field limit



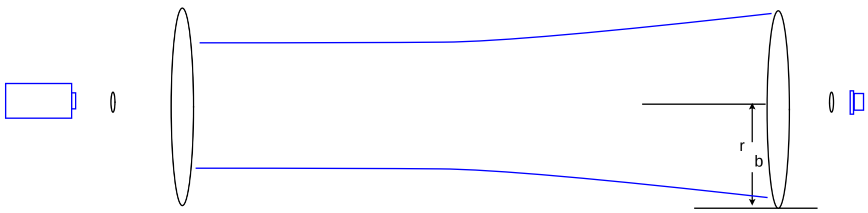
Gaussian beam diffraction:

$$w(z) \approx \frac{\lambda z}{\pi w_0}, \; z \gg z_0 = \frac{\pi w_0^2}{\lambda}, \qquad (1)$$

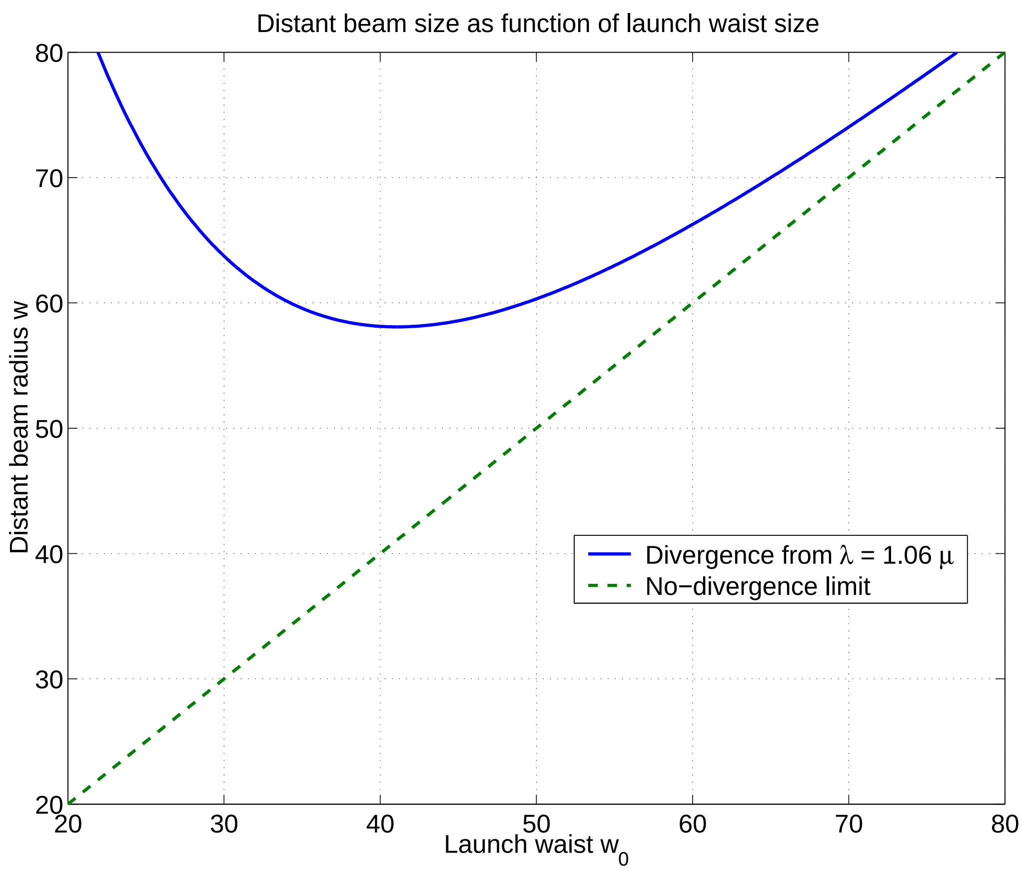
$w_0 =$ beam radius at launch $\approx r_t$, radius of telescope aperture. $w(z) \approx r_b$, beam radius after propagating distance z.

Laser wavelength	λ	1.06 μm
Telescope half-aperture	r_t	15 cm
Propagation distance	z	$5 \cdot 10^9$ m
Divergence half-angle	$\theta = \lambda / \pi r_t$	2 μrad
Rayleigh range	z_0	67 km
One-way Beam size	r_b	11 km
One-way attenuation	$\left(\frac{r_t}{r_b}\right)^2$	$2 \cdot 10^{-10}$
Two-way attenuation	$\left(\frac{r_t}{r_b}\right)^4$	$4 \cdot 10^{-20}$

Big mirrors to catch all the light?



$$w^2(w_0, z) = w_0^2 \left(1 + \left[\frac{z}{z_0(w_0)}\right]^2\right) \tag{2}$$



$w(w_0, z)$ is minimum at $z_0 = z$; minimum value = $\sqrt{2}w_0$.

Ans: mirror diameter $\approx 2w_{\text{min}} \approx 116$ m.

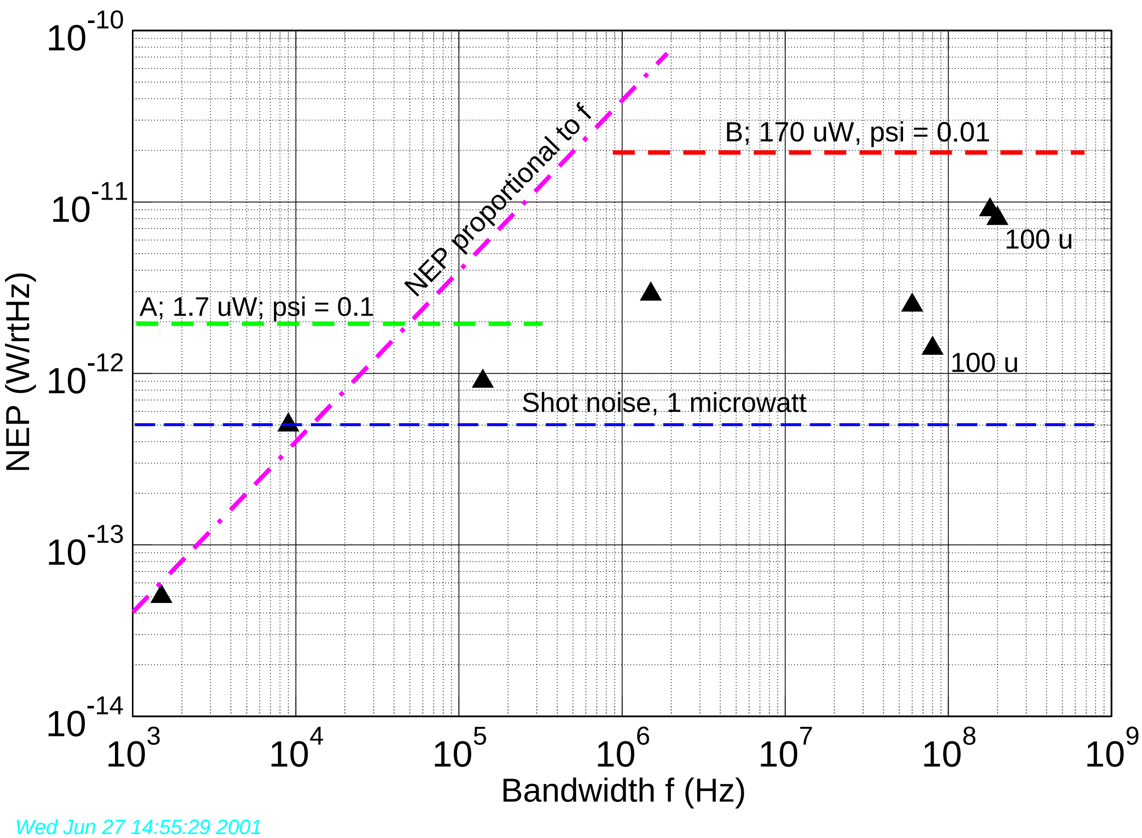
Photodetection

Intensity of 1 W beam at $L = 5 \cdot 10^9$ km is 1 nW/m^2 (stellar magnitude 3.5, not in the top 100 stars)

Detected intensity $P_0 \approx 100$ pW. Compared to shot noise,

$$\tilde{P}_{\text{shotnoise}} = \sqrt{2P_0 h\nu} = 6 \cdot 10^{-15} \text{W}/\sqrt{\text{Hz}} \tag{3}$$

Photodiode/Amplifiers from Analog Modules



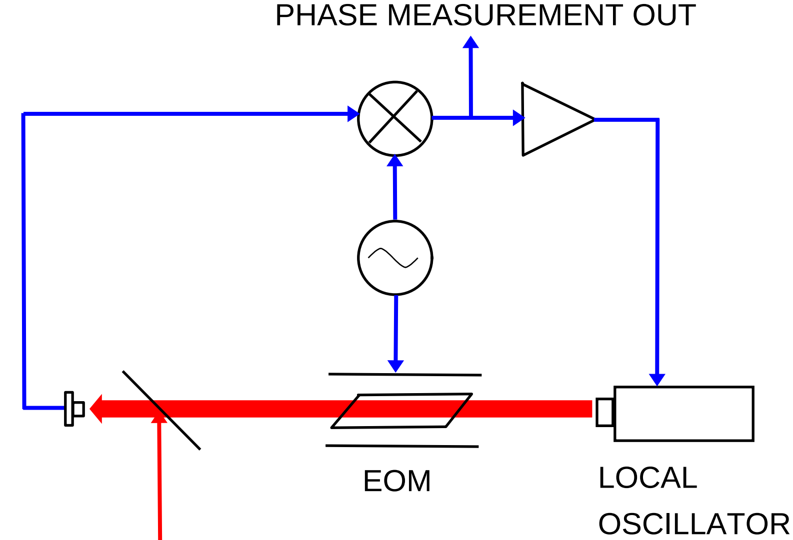
Amplification of weak beam by local oscillator

Amplify before photodetection with local oscillator.

$$I = |E_s + E_L|^2 = |E_s|^2 + |E_L|^2 + 2\Re[E_s^* E_L]. \qquad (4)$$

Cross-term contains phase-sensitive component, $2E_s E_L \cos\phi$.

Modulate: $\phi = \phi_0 + \Gamma \cos \omega t$.

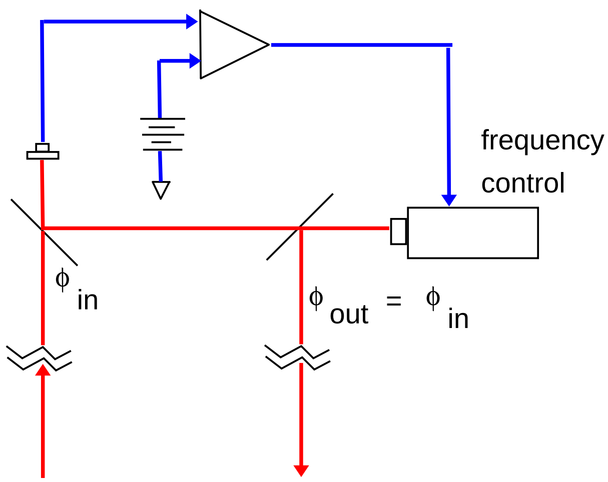


$$\begin{aligned} \cos\phi &= \cos\phi_0 J_0(\Gamma) + 2J_2(\Gamma)\sin 2\omega t + \ldots & (5) \\ &- \sin\phi_0 \left[2J_1(\Gamma)\sin\omega t + 2J_3(\Gamma)\sin 3\omega t + \ldots\right] & (6) \end{aligned}$$

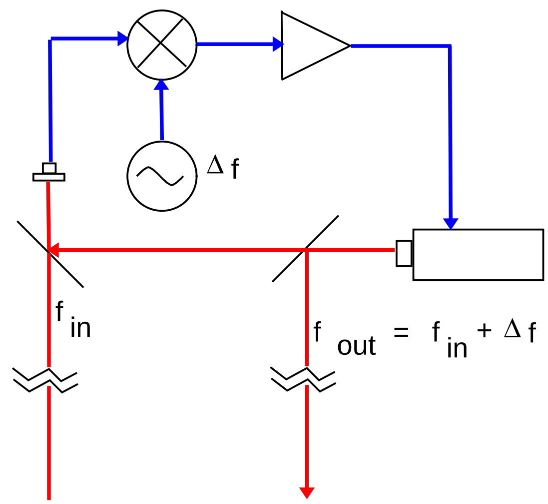
Demodulation measures $\sin\omega t$ component, $\propto \sin\phi_0$, and drives ϕ_0 to zero.

Transponder implementations

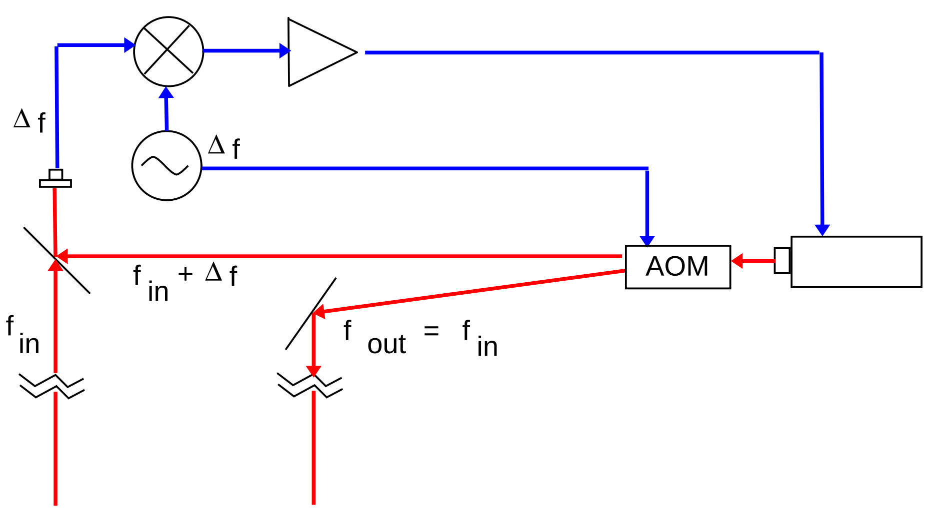
DC LOCK



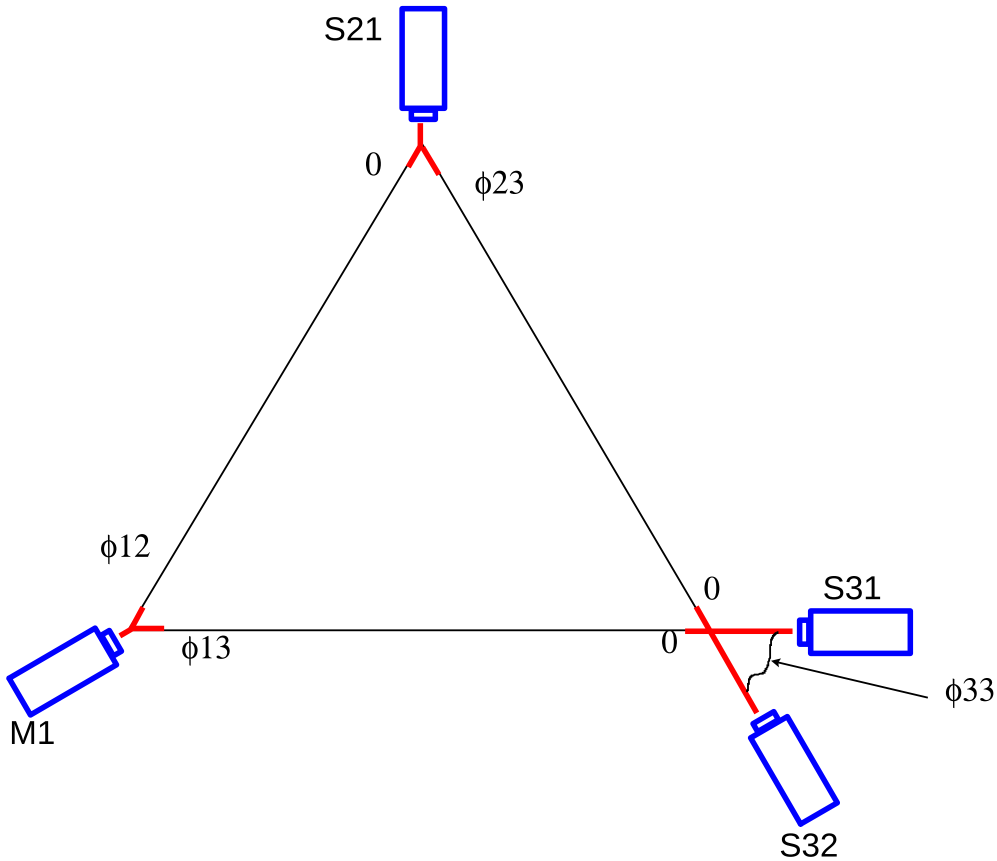
FREQUENCY OFFSET LOCK



OFFSET−CANCELLED LOCK
(D. Shaddock, 5/2002)



Three interferometers, with transponders

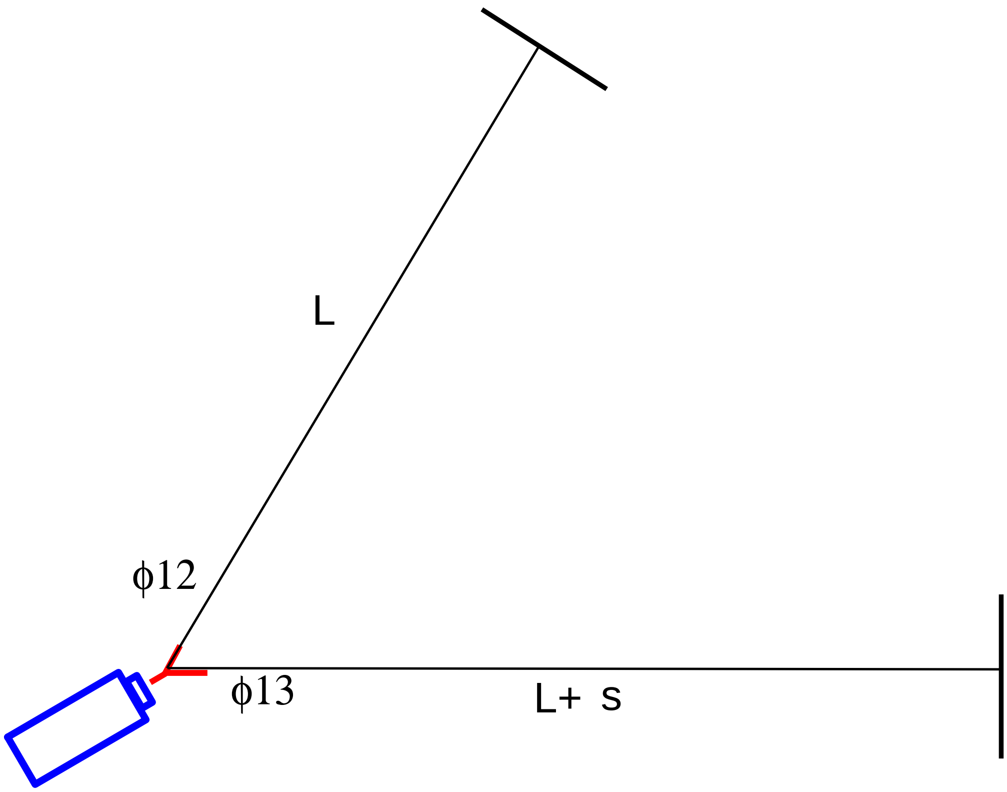


3 independent 2-arm interferometers

1 master laser, 3 slaves, 4 phase measurements

Arm 1 / Arm 2	Phase combination
12/13	$\phi_{12} - \phi_{13}$
12/23	$\phi_{12} - \phi_{23}$
23/13	$\phi_{23} - \phi_{13} - \phi_{33}$

The effect of laser frequency noise



Long-wave limit ($\lambda_{\mathrm{GW}} > L$):

$$h_{\mathrm{signal}} \approx \frac{\Delta L_{\mathrm{signal}}}{L} = \frac{1}{L}\frac{\lambda_o}{2\pi}(\phi_2 - \phi_1) \approx 10^{-21}/\sqrt{\mathrm{Hz}} \quad (7)$$

Laser frequency noise Δf: $\Delta L_{\mathrm{noise}}/s = \Delta f / f_o$, so

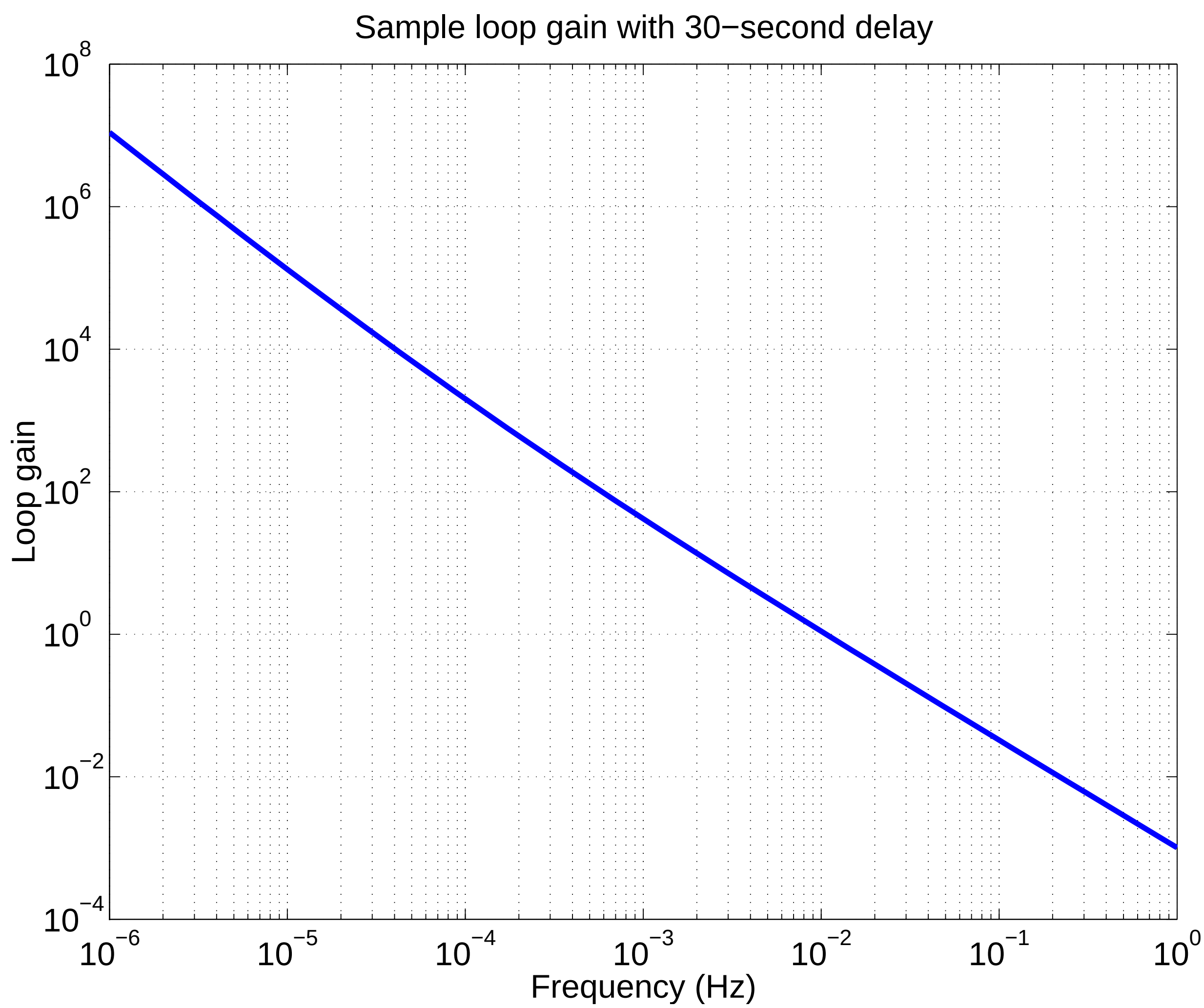
$$h_{\mathrm{noise}} = \frac{s}{L}\frac{\Delta f}{f_o} \approx 10^{-2} \cdot \frac{30\,\mathrm{Hz}\sqrt{\mathrm{Hz}}}{3 \cdot 10^{14}\,\mathrm{Hz}} = 10^{-15}/\sqrt{\mathrm{Hz}} \quad (8)$$

Suppress or subtract Δf by single-arm measurement, ϕ_{12}:

$$\Delta f = \frac{1}{2\pi}\frac{d}{dt}\phi_{12} \quad (9)$$

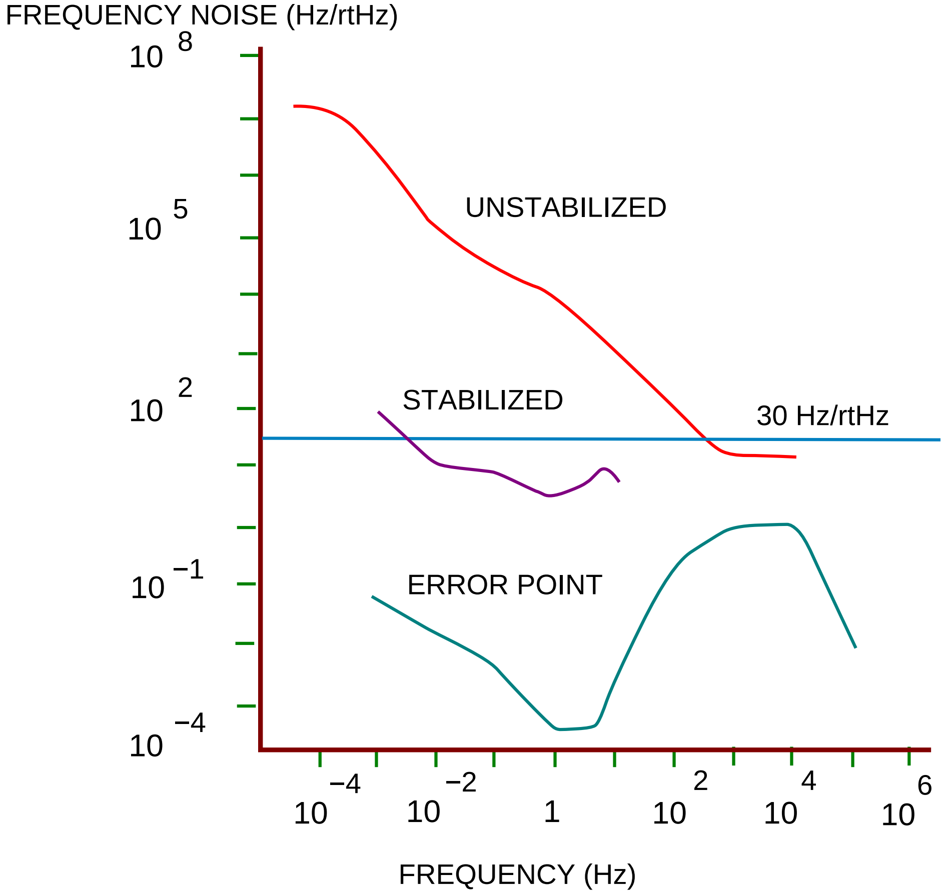
Can feed back to laser frequency to drive Δf to zero.

Propagation delay as limit to control system gain

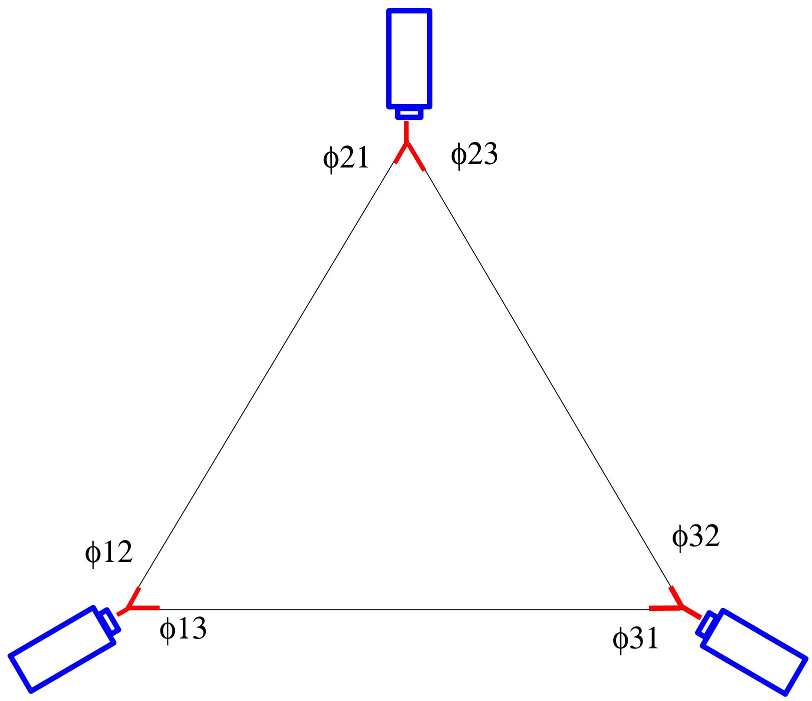


Maximum loop gain at 1 mHz: $G(f) \approx 50$.

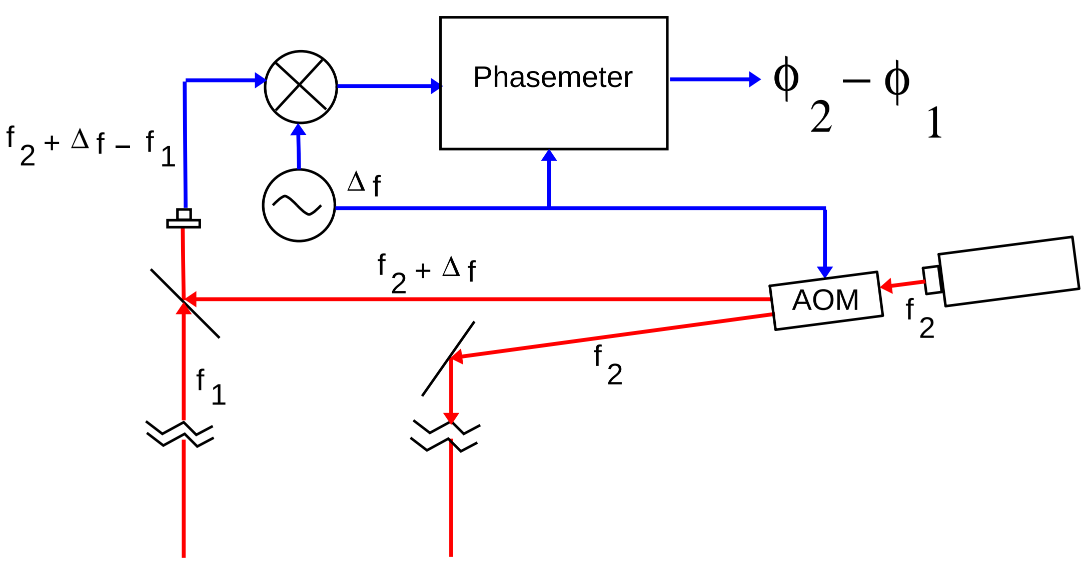
Laboratory demonstration of laser frequency stability



Three interferometers, free-running lasers



Phase-comparison measurement:



G.W. signals from $\phi_{jk}(t)$ by *Time-Delay Interferometry*.

Time-shifting as a linear filter

Subtract two signals with timing error t_0:

$$x(t) = y(t) - y(t - t_0). \tag{10}$$

Fourier Transform:

$$\begin{aligned} X(\omega) &= Y(\omega) - Y(\omega)e^{-i\omega t_0} = Y(\omega)(1 - e^{-i\omega t_0}) \quad (11)\\ &= H(\omega)Y(\omega). \quad (12) \end{aligned}$$

$H(\omega) = 1 - \exp(-i\omega t_0)$ is a linear filter. In general, a filter H multiplies PSD by $|H|^2$:

$$S_{t_0}(\omega) = S_y(\omega)|H(\omega)|^2 = S_y(\omega)2(1 - \cos(\omega t_0)), \tag{13}$$

or

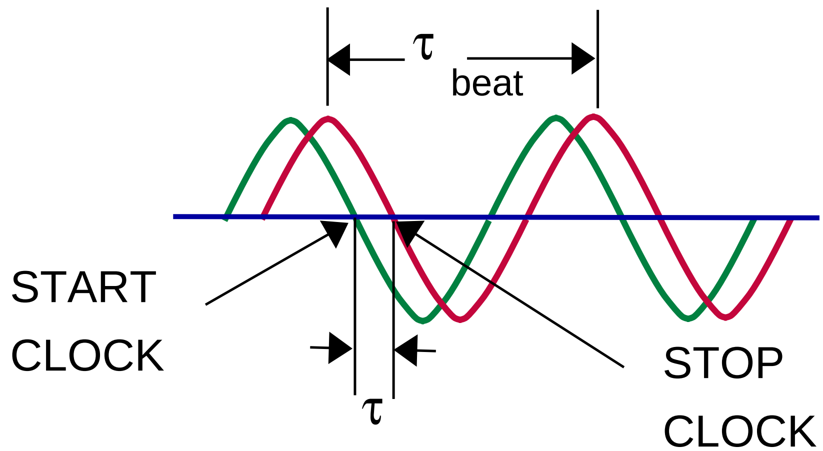
$$S_{t_0}(f) = S_y(f)(1 - \cos(2\pi f t_0)) \approx S_y(f)(2\pi f t_0)^2 \tag{14}$$

$$\tilde{h}_{t_0}(f) \approx \tilde{h}_y(f)(2\pi f t_0) \tag{15}$$

$2\pi f t_0 \approx 10^{-9}$ for $t_0 = 100$ nsec and $f = 1$ mHz.

Absolute metrology to 30 m is adequate to compensate for the effect of completely imbalanced frequency noise.

How a phasemeter works



$$\phi = 2\pi \frac{\tau}{\tau_{\rm beat}} = 2\pi\tau f_b \tag{16}$$

τ measured by clock with frequency f_c: count n ticks of clock:

$$\tau = n/f_{\rm c}. \tag{17}$$

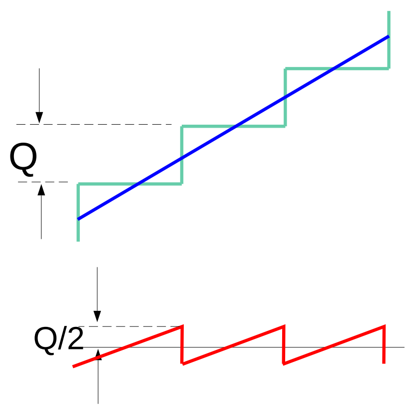
Time quantization: $Q_\tau = 1/f_c$. The corresponding RPSD of τ:

$$\tilde{\tau}(f) = \frac{Q_\tau}{\sqrt{12 f_b}}, \tag{18}$$

or

$$\tilde{\phi}(f) = 2\pi\tilde{\tau}(f) f_b = \frac{2\pi}{f_c}\sqrt{\frac{f_b}{12}} \tag{19}$$

Spectrum of quantization noise



Mean-square error σ^2 is average of sawtooth-squared over one cycle:

$$\sigma^2 = \frac{1}{T}\int_{-T/2}^{T/2}\left(\frac{Qt}{T}\right)^2 dt = \frac{Q^2}{12} \quad (20)$$

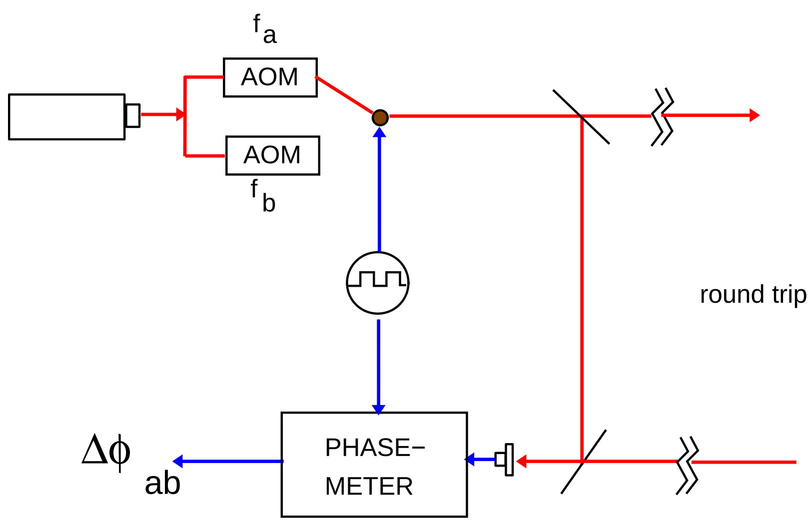
Error at sample rate f_s, white from 0 to $f_s/2$; i.e. Power spectrum $S_V(f)$ is constant. $S_V(f)$ integrates to σ^2.

$$\sigma^2 = \int_0^{f_s} S_V(f)df, \quad (21)$$

or

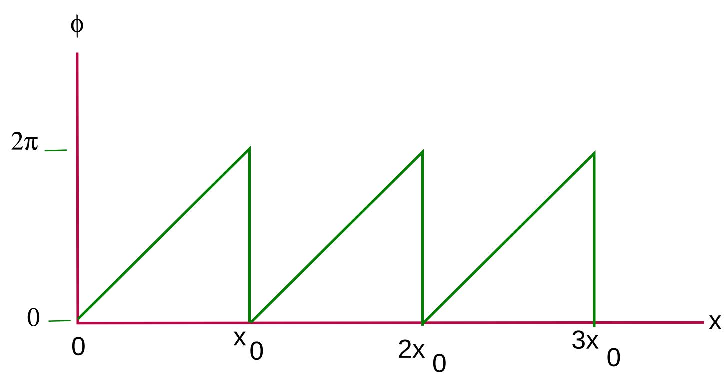
$$S(f) = \frac{Q^2}{12 f_s} \quad (22)$$

Absolute metrology of arm length



$$\Delta\phi_{ab} = 2\pi(n_{\text{cy}} \bmod 1) \tag{23}$$

$$n_{\text{cy}} = n_a - n_b = L/\lambda_a - L/\lambda_b \tag{24}$$

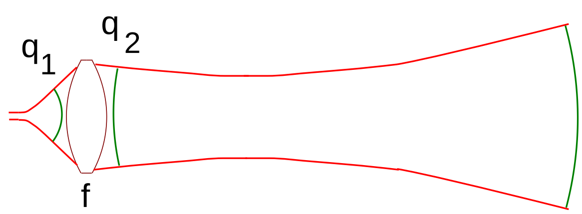


Ambiguity range x_0:

$$x_0 = 1/(1/\lambda_a - 1/\lambda_b) = \frac{c}{f_a - f_b} \tag{25}$$

Gaussian optics for telescope

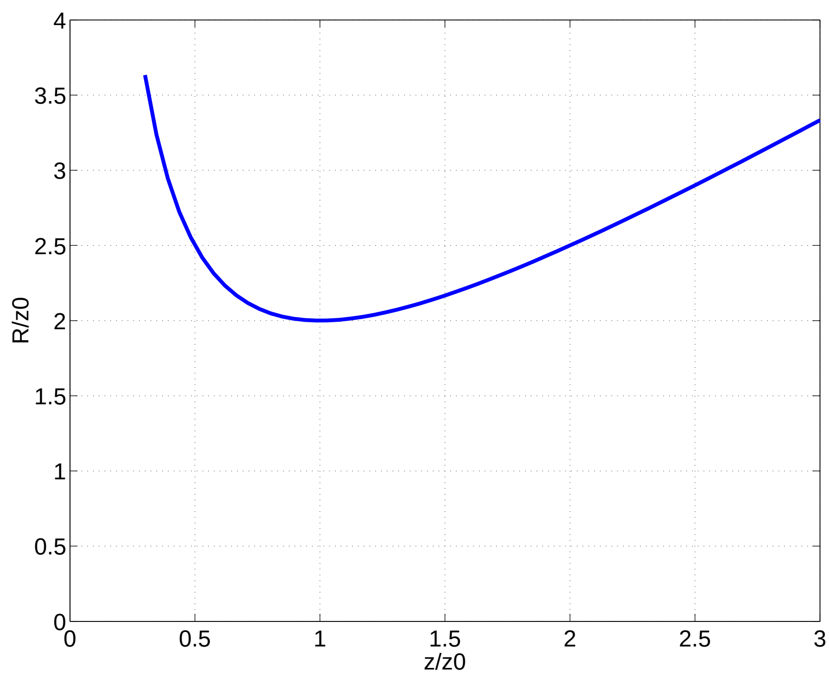
Laser beam focused by telescope with f matched to R:



$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f}, \tag{26}$$

beam radius $= w$; wavefront radius $= R$; $\frac{1}{q} = \left(\frac{1}{R} - i\frac{\lambda}{\pi w^2}\right)$.

$$R(z) = z\left(1 + (z_0/z)^2\right) \tag{27}$$



If $f = R_1 + \varepsilon$, Eq. 26 gives

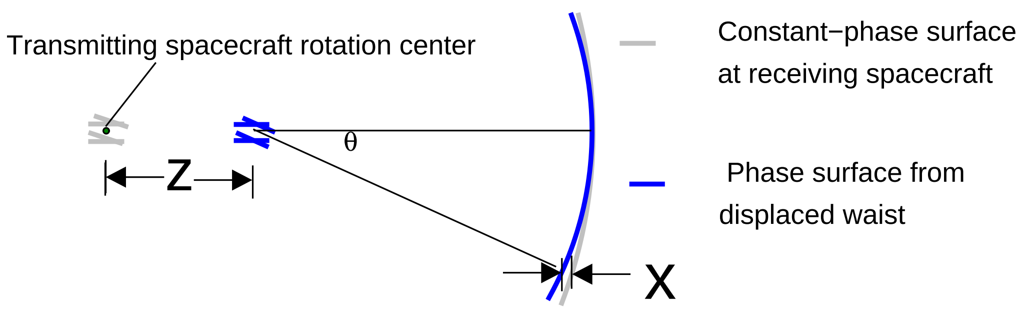
$$1/R_2 = 1/R_1 - 1/(R_1 + \varepsilon) \approx \varepsilon/R_1^2. \tag{28}$$

Eq. 27 then gives z, displacement of waist:

$$z = z_0^2 \frac{1}{R_2} \approx z_0^2 \varepsilon / R_1^2 = \left(\frac{\pi w_1^2}{\lambda}\right)^2 \frac{\varepsilon}{R_1^2}. \tag{29}$$

$w_1 = D/2; \qquad R_1 = f = Df^{\#} \approx D$, so

$$z = \left(\frac{\pi D}{4\lambda}\right)^2 \approx 5\,\mathrm{km} \tag{30}$$



Signal error $x = z\cos\theta \approx -z\theta\Delta\theta$, or $\tilde{x}(f) = z\tilde{\theta}(f)\theta_{DC}$. For $\tilde{\theta}(f) = 10\,\mathrm{nrad}$, $\tilde{x}(f) < 1\,\mathrm{pm}/\sqrt{\mathrm{Hz}}$ requires

$$\tilde{\theta}\theta_{DC} < 200\,\mathrm{nrad}^2. \tag{31}$$